

Name of Course	: B.Sc. (Math. Sci.)-I, B.Sc. (Phy. Sci.)-I, B.Sc. (Life Sci.)-I
Unique Paper Code	: 42351101_OC
Name of Paper	: Calculus and Matrices
Semester	: I
Duration	: 3 hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

Qs 1: Is set $S = \{(1, 2, 0), (0, 1, 2), (1, 0, 2)\}$ a basis of $\mathbb{R}^3$?

Let $P = \{(a, b): a, b \in \mathbb{R}, a + b \geq 0\}$ and $Q = \{(a, b): a, b \in \mathbb{R}, b = 4a + 10\}$

Are P and Q subspaces of $\mathbb{R}^2$?

Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by $T(x, y) = (x + y, x - 7y)$

Show that T is a linear transformation and find the matrix representation of T .

Qs 2: Find the inverse of the matrix P , if it exists, using elementary row operations where

$$P = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the rank of the matrix B where

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 2 & 3 & 0 \\ 4 & 6 & 0 \end{bmatrix}$$

Using elementary row operations, find the solution of the following system of equations, if it exists

$$x + y + z = 2$$

$$2x - z = 0$$

$$3x + y + z = 4$$

Qs 3: Find the eigen values of matrix A and corresponding eigen vectors for each of the eigen values of A where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the limit of the sequence, if it converges:

a. $\langle 10 + \frac{n+1}{n^2+1} \rangle$

b. $\langle (-1)^{n+1} \frac{\sin n}{n^2} \rangle$

Let $f(x) = x^3 - 3x + 1$. Answer the following for $f(x)$:

- (i) Is $f(x)$ symmetric about y - axis?
- (ii) Find formula to new function which is obtained after translating $f(x)$ by 4 units to the left.

Qs 4. Assuming the validity of expansion, find the Taylor series generated by $f(x) = e^{2x}$ at $x = 0$.

If $y = \tan^{-1} x$, show that $(1 + x^2)y_{n+2} + 2(n + 1)xy_{n+1} + n(n + 1)y_n = 0$.

Find the second-order partial derivatives of $f(x, y) = x^3 + y^3 - 2x^2y^2$.

Qs 5. Verify that $z(x, y) = e^{-x} \sin y$ is a solution of the Laplace equation.

Find $|z|, \arg z, \arg \bar{z}, \arg(-z)$ for the complex number $z = (1 - \sqrt{3}i)(-1 - i)$.

Find the equation of the circle described on the line joining the points given by $1 + 2i$ and $5 - 6i$ as diameter.

Qs 6. Form an equation of lowest degree with real coefficients whose roots are $1, -1, i$.

If n is a positive integer, then show that $(1 + i)^n - (1 - i)^n = i 2^{\frac{n}{2}+1} \sin(\frac{n\pi}{4})$.

A body with initial temperature of 120°C is allowed to cool in air which remains at a constant temperature of 30°C . It is given that after 10 minutes, the body has cooled to 60°C . Find the temperature of the body after 20 minutes.